

PRIORITY RESEARCH CENTRE
COMPUTER ASSISTED RESEARCH
IN MATHEMATICS AND ITS APPLICATIONS



*Exploratory Experimentation and Computation
in Number Theory Workshop*

7–9 July 2010

Wednesday 7 July

Biranks for partitions into 2 colors and some theta function identities

Francis G. Garvan

Department of Mathematics, University of Florida, USA

The number of 2-colored partitions of n is congruent to $0 \pmod{5}$ for n congruent to 2, 3, or 4 mod 5. In 2003, Hammond and Lewis gave a statistic called the birank which divides these partitions into 5 equal classes. We give two deeper analogs. One analog is in terms of Dyson's rank and the second uses the 5-core crank due to Garvan, Kim and Stanton. The Dyson rank analog follows from an identity due to Ramanujan. The 5-core crank analog follows from a certain multidimensional theta function identity, which we prove with the help of MAPLE. If time permits, we describe and demonstrate a new MAPLE package for proving theta-function identities.

Finite analogues of Rogers–Ramanujan type identities

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The first Rogers–Ramanujan identity can be stated as follows: The partitions of an integer n in which the difference between any two parts is at least 2 are equinumerous with the partitions of n into parts congruent to 1 or 4 modulo 5.

Gordon generalized this result from modulo 5 to one involving arbitrary odd k and an analytic form was found by Andrews. Since the pioneering work of Andrews, many infinite families of identities of similar type have been found.

In this talk, we will consider some polynomial analogues of these Rogers–Ramanujan type identities.

¹Joint work with Ole Warnaar.

Thursday 8 July

Ramanujan and the theory of modular forms

Heng Huat Chan

Department of Mathematics, National University of Singapore, Singapore

In this talk, I will first introduce the Indian Mathematician Srinivasa Ramanujan. I will then present several results of Ramanujan's mathematics that have close connections with the theory of modular forms.

Ramanujan and Euler's constant

Richard P. Brent

Australian National University, Canberra

In memory of Edwin M. McMillan

Corollary 2, Entry 9, Chapter 4 of Ramanujan's first notebook claims that

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{nk} \left(\frac{x^k}{k!}\right)^n \sim \ln x + \gamma \quad \text{as } x \rightarrow +\infty,$$

where x is a real variable, n is a positive integer parameter, and $\gamma \approx 0.577$ is Euler's constant. Ramanujan's claim is known to be correct if $n \geq 2$, but incorrect if $n > 2$. We consider the order of the error term, and discuss a different (correct) generalisation of the case $n = 1$. The generalisation is useful for high-precision computation of γ .

Higher-rank Rogers–Ramanujan identities

Ole Warnaar

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The classical Rogers–Ramanujan (RR) identities are some of the most famous identities in additive number theory. Apart from their number theoretic contents, they also have deep connections with the representation theory of quivers and infinite-dimensional Lie algebras. I will use this connection to argue that the RR identities should extend to all simple-laced Lie algebras. To actually obtain these extensions requires either divine inspiration or extensive exploratory experimentation and computation.

No knowledge of quivers or infinite-dimensional Lie algebras will be assumed in this talk.

Apéry sequences and series for $1/\pi$

Shaun Cooper

Massey University, Auckland, New Zealand

One of the sequences discussed by Apéry in his 1978 lecture on the irrationality of $\zeta(3)$ is given by the recurrence relation

$$n^2 u_n = (11n^2 - 11n + 3)u_{n-1} + (n-1)^2 u_{n-2}, \quad n = 2, 3, \dots,$$

and initial conditions $u_0 = 1$, $u_1 = 3$. Remarkably, all terms in this sequence are integers.

We will present 4 series for $1/\pi$ that involve the numbers u_n . These are analogues for $\Gamma_0(5)$ of series discovered recently by Chan, Tanigawa, Yang and Zudilin for $\Gamma_0(6)$. The corresponding series for $\Gamma_0(8)$ (5 series) and $\Gamma_0(9)$ (8 series) will also be discussed.

Rigidity for homomorphisms from arithmetic groups to locally compact groups

George Willis¹

CARMA, The University of Newcastle, NSW

The Margulis Superrigidity Theorem applies to homomorphisms from arithmetic groups to Lie groups. Superrigidity may be strengthened (for groups such as $SL(n, \mathcal{O})$ with either $n \geq 3$ or \mathcal{O} an algebraic integer ring with infinitely many units) by significantly expanding the class of homomorphisms to which it applies. This strengthened theorem, which is joint with Yehuda Shalom, is proved by a reduction to Margulis' work that relies on techniques from several distinct areas.

One technique used is the **scale function** on a totally disconnected locally compact group. This positive integer-valued function carries information about the structure of the group. The talk will explain what this function is and how it is used in the extension of Margulis' Theorem.

On group structures realised by elliptic curves over a fixed finite field

Reza Rezaeian Farashahi

Macquarie University, Sydney, NSW

We obtain explicit formulas for the number of non-isomorphic elliptic curves with a given group structure (considered as an abstract abelian group). Moreover, we give explicit formulas for the number of distinct group structures of all elliptic curves over a finite field. We use these formulas to derive some asymptotic estimates and tight upper and lower bounds for various counting functions related to classification of elliptic curves accordingly to their group structure. Finally, we present results of some numerical tests which exhibit several interesting phenomena in the distribution of group structures.

¹Joint work with Yehuda Shalom.

Friday 9 July

The arithmetic of uniform random walks

Jonathan Borwein¹

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Following Pearson in 1905, we study the expected distance of a two-dimensional walk in the plane with n unit steps in random directions — what Pearson called a “ramble”. While the large n behaviour is well understood, the precise behaviour of the first few steps is quite remarkable and less tractable. Series evaluations and recursions are obtained making it possible to explicitly determine this distance for small number of steps.

Hypergeometric and elliptic closed form expressions are given for all the moments of a 2- or 3-step walk and of a 4-step walk. Heavy use is made of the analytic continuation of the underlying integral (also of special functions and computer algebra (CAS)). This is joint work with Dirk Nuyens, Armin Straub and James Wan. A related paper is at: <http://www.carma.newcastle.edu.au/~jb616/walks2.pdf>.

Probability densities of random walks

James Wan

CARMA, The University of Newcastle, NSW

Over 100 years ago Pearson asked for the probability density of a uniform random walk on the plane after n steps. It turns out the behaviour for a small number of steps is quite different from, and more interesting than, the behaviour after a large number of steps. Though little progress had been made on this problem, this talk will highlight some features of 3- and 4-step walks densities, some of which were discovered by experimentation.

¹Joint work with Dirk Nuyens, Armin Straub and James Wan.

Series for $1/\pi$ revisited

Wadim Zudilin¹

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I will discuss certain impractical generalisations of Ramanujan's series for $1/\pi$: divergent, complex and congruence evaluations.

¹Joint work with Jesús Guillera.