

# APCO Workshop Program

Date/Time: Saturday, 17 December 2016/9:20am-5pm

Venue: UNH419, University House, University of Newcastle City Campus

**Opening: 9:20am-9:30am**

**Time: 9:30am-10:20am**

*Plenary Speaker:* Jerzy A. Filar

School of Mathematics and Physics, The University of Queensland, Australia

*Title:* Controlled Markov chains and the Hamiltonian cycle problem

*Abstract:* We consider the famous Hamiltonian cycle problem (HCP) embedded in a Markov decision process (MDP). More specifically, we consider a moving object on a graph  $G$  where, at each vertex, a controller may select an arc emanating from that vertex according to a probabilistic decision rule. A stationary policy is simply a control where these decision rules are time invariant. Such a policy induces a Markov chain on the vertices of the graph. Therefore, HCP is equivalent to a search for a stationary policy that induces a 0 - 1 probability transition matrix whose non-zero entries trace out a Hamiltonian cycle in the graph. A consequence of this embedding is that we may consider the problem over a number of, alternative, convex - rather than discrete - domains. These include: (a) the space of stationary policies, (b) the more restricted but, very natural, space of doubly stochastic matrices induced by the graph, and (c) the associated spaces of so-called occupational measures". This approach to the HCP has led to both theoretical and algorithmic approaches to the underlying HCP problem. In this presentation, we outline a selection of results generated by this line of research.

**Time: 10:30am-10:55am**

*Contributed Speaker:* Sogol Mohammadian

School of Mathematical and Physical Sciences, The University of Newcastle, Australia

*Title:* Hamilton cycles, feasible bases and random walk

*Abstract:* In 2000, Feinberg presented a new polytope corresponding to a given graph  $G$ . This mapped Hamiltonian cycles into a subset of extreme points of this polytope for a given graph  $G$ , which creates an opportunity to search for Hamilton cycles among extreme points of the polytope. This fact motivated us to study the structure of feasible bases of this polytope. To this end, we characterized feasible bases of the polytope. Furthermore, we propose to develop a fully polynomial almost uniform sampler to generate feasible bases.

**Morning Tea: 11:00am-11:20am**

**Time: 11:20am-12:10pm**

*Plenary Speaker:* Dana Randall

College of Computing, Georgia Institute of Technology, US

*Title:* Slow mixing of Markov chains for sampling independent sets

*Abstract:* For many combinatorial models on  $Z^d$ , the natural sampling algorithms based on local Markov chains work at sufficiently low density (or high temperature) and break down at high density (or low temperature). We will discuss this phenomenon in the context of independent sets on  $Z^2$ , also known as the hard-core lattice gas model in statistical physics. For a given  $\lambda$ , the Gibbs distribution is defined so that each independent set  $I$  has probability proportional to  $\lambda^{|I|}$ . We are interested in determining the mixing time of local Markov chains that add or remove a small number of vertices in each step. On finite regions of  $Z^2$  it is conjectured that there is a phase transition at some critical point  $\lambda_c$  that is approximately 3.79 such that below this value local chains converge quickly, while above they are prohibitively slow. Fast mixing has been established for  $\lambda < 2.48$ . We give complementary results showing that local chains mix slowly when  $\lambda > 5.365$  on regions with periodic (toroidal) boundary conditions and  $\lambda > 7.103$  on regions with non-periodic (free) boundary conditions. The proofs are based on combinatorial insights that characterize independent sets by the presence or absence of large dense clusters.

**Time: 12:20pm-12:45pm**

*Contributed Speaker:* Mikhail Isaev

School of Mathematics and Statistics, The University of New South Wales, Australia

*Title:* Cumulant expansion in combinatorial enumeration

*Abstract:* Many enumeration problems in combinatorics, including such fundamental questions as the number of regular graphs, can be expressed as high-dimensional complex integrals. The asymptotic behaviour is found by concentrating the integral in a small region and then approximating the integrand inside that region. Typically, the integrals that occur are of the form

$$I = \int_{\Omega} \exp(-\mathbf{x}^T A \mathbf{x} + f(\mathbf{x})) d\mathbf{x}.$$

where  $A$  is a positive-definite real matrix and  $f(\mathbf{x})$  is a polynomial of low degree with complex coefficients.

Let  $\mathbf{X}$  be a random variable with distribution given by the gaussian density  $C \exp(-\mathbf{x}^T A \mathbf{x})$  truncated to domain  $\Omega$ . Then we have  $I = C^{-1} \mathbb{E} e^{f(\mathbf{X})}$ , so the problem is reduced to estimating  $\mathbb{E} e^{f(\mathbf{X})}$ .

All relevant examples previously considered in the literature are based on estimates equivalent to  $\mathbb{E} e^{f(\mathbf{X})} \sim e^{\mathbb{E} f(\mathbf{X})}$  or  $\mathbb{E} e^{f(\mathbf{X})} \sim e^{\mathbb{E} f(\mathbf{X}) + \frac{1}{2} \mathbb{V} f(\mathbf{X})}$ . It is a natural idea to employ the cumulant series for the purpose of obtaining more precise bounds. However, the presence of the imaginary part of  $f$  does not allow us to go beyond the second-order approximation in general. In this talk we discuss additional conditions on  $f(\mathbf{X})$  which suffice to bypass this obstacle. These conditions frequently hold in combinatorial problems.

**Lunch: 12:50pm-2:00pm**

**Time: 2:00pm-2:50pm**

*Plenary Speaker:* Nick Wormald

Department of Mathematical Sciences, Monash University, Australia

*Title:* A natural infection model

*Abstract:* Suppose that individuals are randomly placed points in space according to a Poisson process, and have two states, infected or healthy. Any infected individual passes the infection to any other at distance  $d$  according to a Poisson process, whose rate is a function  $f(d)$  of  $d$  that decreases with  $d$ . Any infected individual heals at rate 1. Initially, one individual is infected. An epidemic is said to occur when the infection lasts forever. We investigate conditions on  $f$  under which the probability of an epidemic is nonzero. This is joint work with Josep Diaz and Xavier Perez Gimenez.

**Time: 3:00pm-3:25pm**

*Contributed Speaker:* Rajko Nenadov

Department of Mathematical Sciences, Monash University, Australia

*Title:* Optimal induced universal graphs for bounded-degree graphs

*Abstract:* We show that for any constant  $\Delta \geq 2$ , there exists a graph  $\Gamma$  with  $\mathcal{O}(N^{\Delta/2})$  vertices which contains every  $n$ -vertex graph with maximum degree  $\Delta$  as an induced subgraph. For odd  $\Delta$  this significantly improves the best-known earlier bound of Esperet et al. and is optimal up to a constant factor, as it is known that any such graph must have at least  $\Omega(n^{\Delta/2})$  vertices.

Our proof builds on the approach of Alon and Capalbo (SODA 2008) together with several additional ingredients. The construction of  $\Gamma$  is explicit and is based on an appropriately defined composition of high-girth expander graphs. The proof also provides an efficient deterministic procedure for finding, for any given input graph  $H$  on  $n$  vertices with maximum degree at most  $\Delta$ , an induced subgraph of  $\Gamma$  isomorphic to  $H$ .

**Afternoon Tea: 3:30pm-3:50pm**

**Time: 3:50pm-4:15pm**

*Contributed Speaker:* Rebecca Stones

College of Computer and Control Engineering at Nankai University, China

*Title:* Being accurate is not enough: New metrics for disk failure prediction

*Abstract:* We propose new metrics for measuring reliability in storage systems with proactive fault tolerance, migration rate and mismigration rate, which account for the time it takes to transfer data after failure prediction. We use these to compare prediction methods and find that traditional methods are not always consistent. These metrics can help designers choose better trade-offs and reduce the transfer costs when failures are predicted.

**Time: 4:20pm-4:45pm**

*Contributed Speaker:* Michael Haythorpe

Flinders Mathematical Sciences Laboratory, Flinders University, Australia

*Title:* Solving discrete combinatorial problems via continuous optimisation techniques

*Abstract:* A wide variety of important, difficult problems can be cast as discrete optimisation problems with linear constraints. It has long been recognised that such problems can be further cast as (nonconvex) continuous optimisation problems with linear constraints, by first relaxing the discrete requirements, and then ensuring the global solution has not been altered through the use of appropriate penalty functions or other such techniques. However, in practice, standard continuous optimisation algorithms usually perform quite poorly on problems posed in this way.

In this talk, I consider one such relaxed formulation based on the Hamiltonian cycle problem, a discrete NP-complete problem. I describe an algorithm which was specifically designed to take advantage of the features present in the formulation. I discuss some interesting features that arise which are likely to be present in all continuous formulations arising from relaxations of discrete optimisation problems.

**Conclusion: 4:50pm-5:00pm**