Let $U$ be an $n$-dimensional vector space over a finite field of $q$ elements. The number of elements of $\Lambda^2 U$ of each irreducible length is found using the isomorphism of $\Lambda^2 U$ with $H_n$, the space of $n \times n$ skew-symmetric matrices, and results due to Carlitz and MacWilliams on the number of skew-symmetric matrices of any given rank.

Let $U$ be an $n$-dimensional vector space over a finite field $F = GF(q)$. We consider the elements of $\Lambda^2 U$ (called bivectors, or 2-vectors). The (irreducible) length of a 2-vector is well known. Any 2-vector can be expressed as a sum $\sum_i x_i \wedge y_i$ where $\{x_1, \ldots, x_r, y_1, \ldots, y_s\}$ is independent and then its length is $r$. The 2-vectors of length 1 are called decomposable.

Of the $q^{n^2}$ elements of $\Lambda^2 U$, it is difficult to count directly the number having a fixed length, since there is no unique representation for a 2-vector as a sum of the minimal number of decomposables. However, we can make use of the isomorphism of $\Lambda^2 U$ with $H_n$, the space of $n \times n$ skew-symmetric matrices over $F$. This isomorphism, denoted $\phi$, is shown by Marcus and Westwick [3] to have the property that $z \in \Lambda^2 U$ has length $r$ if and only if $\phi(z) \in H_n$ has rank $2r$. The number of skew-symmetric matrices of rank $2r$ has been determined by Carlitz [1] and MacWilliams [2]. Consequently, we have

**Theorem.** If $U$ is a vector space of dimension $n$ over $GF(q)$, the number of vectors in $\Lambda^2 U$ of length $r$ is

$$K(n, r) = \prod_{i=1}^{r} \frac{q^{2i-2} - 1}{(q^{2i} - 1)} \prod_{i=0}^{2r-1} (q^{n-1} - 1)$$

This is valid even when $q = 2^t$, although then $\Lambda^2 U$ coincides with the symmetric product $V^2 U$. 

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